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Department of Mathematics
Technical University of Munich

Algorithms in Uncertainty Quantification

Kickoff UNMIX project

Mario Teixeira Parente
Department of Mathematics
Chair of Numerical Mathematics
E-mail: `parente@ma.tum.de`
Web: `http://www.mateipa.de`

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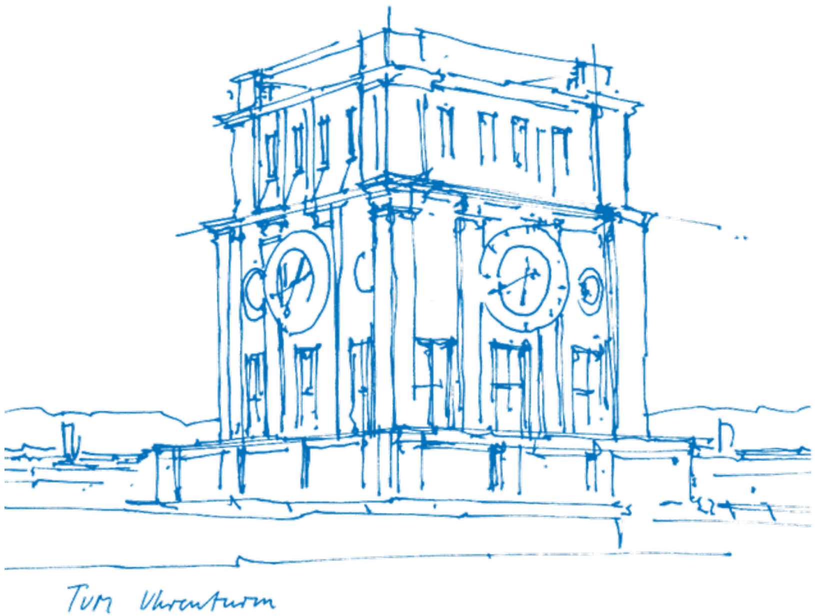

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Uncertainty Quantification

Physical models are subject to uncertainties of different kinds/sources[1]:

- Model error
- Measurement noise
- Discretization error
- **Parameter uncertainty**
- Uncertainty in the system of reasoning

[1] [Oden, 2017]

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Parameter inference

Task: Find the “best“ model parameters θ that explain measured data d.

$\rightarrow$ Inverse problem: Find $\theta^\star$ such that

$$d = \mathscr{G}(\theta^\star) \qquad \text{or} \qquad \theta^\star = \arg\min_\theta \|d - \mathscr{G}(\theta)\|_2.$$

$\rightarrow$ **ill-posed** (and no uncertainties)

$\rightarrow$ Inference in a probabilistic framework[2]: **Bayesian inversion**

[2] [Tarantola, 2005, Kaipio and Somersalo, 2006]

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Bayesian inversion

Idea: Treat data and parameters as random variables.

Assume noise in measurements: $\eta \sim \mathcal{N}(0, \Gamma)$

$$d = \mathcal{G}(\theta) + \eta \tag{1}$$

Task: Find *posterior distribution* $\rho(\theta|d) \propto \rho(d|\theta)\rho(\theta)$.

Analytical expression for the posterior are prohibitive. $\rightarrow$ Create samples

If one forward solve has high computational cost and number of dimensions is non-trivial, then sampling is very expensive. $\rightarrow$ Surrogate models, dimension reduction

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Dimension reduction with *active subspaces*

Approximate a high-dimensional function $f : \mathbb{R}^n \to \mathbb{R}$ with a lower-dimensional function $g : \mathbb{R}^k \to \mathbb{R}$ $(k < n)$ by concentrating on “important directions“ in the domain.

Interpretation in Bayesian inversion:
Infer only those parameters (more accurate: directions in the parameter space) that are informed by data.

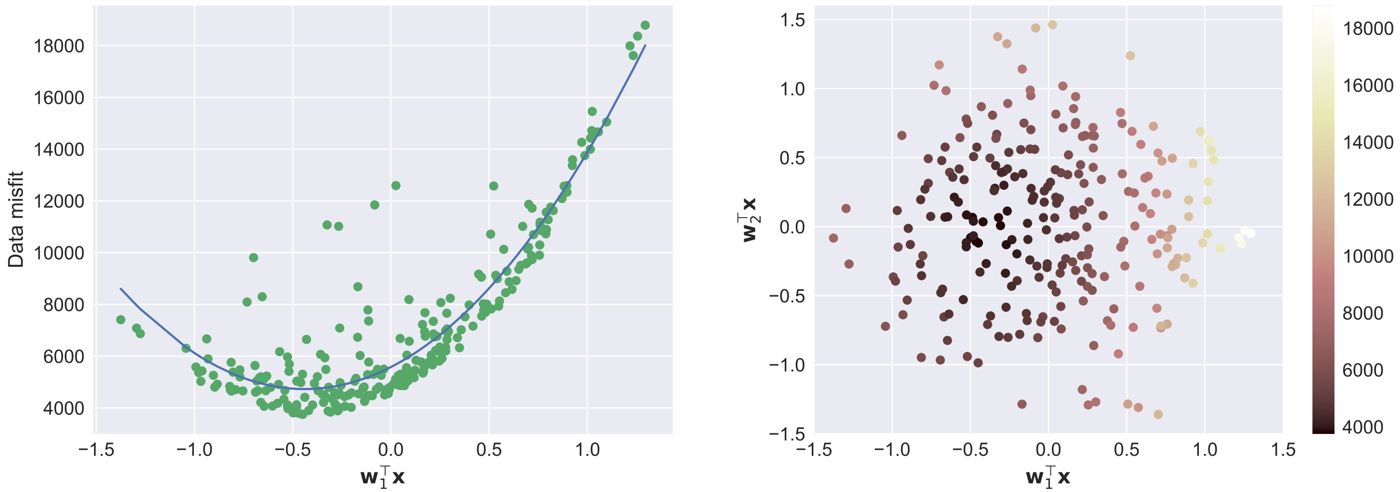

Fig.: Data misfit of an 8D parameter space plotted on the most important axes [Teixeira Parente et al., 2018].

Future investigations

- Avoiding MCMC with *transport maps*
- Sparse grids in the parameter space
- Reduced basis approach for Bayesian inversion

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Potential topics for (Master) theses

- **Reduced basis approach for Bayesian inversion**
  Shift expensive computations to a offline-phase and use results to accelerate online computations.

- ***Consistent* Bayesian formulation of stochastic inverse problems**[3]
  Combine a measure-theoretic approach to stochastic inverse problems with the conventional Bayesian formulation. Use new ideas to lower the influence of the prior on the posterior.

[3] [Butler et al., 2017]

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